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THE ARABIC TEXTUAL TRADITIONS OF EUCLID'S ELEMENTS

BY GREGG DE YOUNG
HISTORY OF SCIENCE,
STONEHILL COLLEGE,
NORTH EASTON, MASSACHUSETTS 02356

SUMMARIES

Recently completed studies of book V and books VII-IX of the Arabic versions of Euclid's Elements *indicate the necessity for a revision of the standard view of the Euclidean tradition in the Islamic world. Newly discovered manuscripts show that we have two distinct versions, each attributed to Isḥāq ibn Ḥunayn (in the revision of Thābit ibn Qurrah) although the received tradition does not distinguish the two. A newly analyzed anonymous translation (now in Leningrad) appears to have ties to the lost translations of al-Ḥajjāj and may provide important information for reconstructing the main features of these lost versions.*

Deux études des traductions arabes des Éléments *d'Euclide récemment completés, une du livre V et l'autre des livres VII-IX, indiquent la nécessité de réviser l'opinion reçue de la tradition euclidienne dans le monde islamique. Des manuscrits recemment découverts indiquent que nous avons deux versions distinctes, chacune attribuée à Isḥāq ibn Ḥunayn (et révisée par Thābit ibn Qurrah), quoique l'opinion reçue n'établisse pas de distinction entre ces deux. Une traduction anonyme, qui se trouve actuallement à Leningrad, et qui a été récemment analysée, paraît avoir des liens aux traductions perdues d'al-Ḥajjāj et pourrait fourni des reseignements importants pour reconstruction des principaux caractéristiques de ces versions perdues.*

The pathways by which the text of Euclid's *Elements* has come to us are among the most convoluted of any literary work. The major features of the transmission that were deduced in the last century through the efforts of Heiberg, Klamroth, Steinschneider, and others have been clearly summarized by T. L. Heath in the introduction to his translation of Heiberg's edition of the Greek text. The discovery of new manuscripts of the Arabic translations of the *Elements* in recent years has shed new light on the Arabic phase of the transmission of the text.

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The most recent studies of the Arabic text of the *Elements* have focused on book V [Engroff 1980] and on books VII-IX [De Young 1981]. These segments of the *Elements* develop the theory of proportions. Book V discusses proportions in the context of continuous magnitudes (i.e., geometric lines) and books VII-IX discuss the same concepts in the context of discontinuous magnitudes (i.e., numbers). The discussion of proportions is preparatory to the study of incommensurable ratios which makes up book X. Both book V and book X have been of special interest to commentators in the Islamic world because they raise interesting philosophical questions, such as how is it possible for two geometrically constructed lengths (each of which can be divided into shorter lengths indefinitely) to be incommensurable?

Before attacking the questions raised in the Arabic commentaries on Euclid, it is essential to establish first the text of the Arabic translations of the *Elements* on which these commentaries are based. These recent studies have attempted to do so in the case of book V and of books VII-IX. In the process, they have raised several issues that require the reevaluation of the translation history. Why, for example, does the translation by Isḥāq ibn Ḥunayn (as revised by Thābit ibn Qurrah) exist in two distinctly different versions? Why are the alternative proofs for propositions VIII-20,21 (which are explicitly attributed to one of the lost translations of al-Ḥajjāj) so different from these proofs in the commentary of al-Nayrīzī, where they are, supposedly, quoted verbatim? And what is the relationship between the newly analyzed anonymous translation (now in Leningrad) and the lost translations of al-Ḥajjāj? Although not all these questions can be answered completely, some preliminary responses can be formulated.

In order to better understand the revisions that these new discoveries suggest, we shall first briefly review the process of transmission from Greek into Arabic as outlined by Heath [1905]. We shall then present new textual evidence indicating the need to revise this accepted view. Finally, we shall indicate how Heath may have been misled by his reliance on Ibn al-Nadīm's biobibliographical study, the *Fihrist* [1969].

The first copies of the Greek text of the *Elements* seem to have reached the Islamic world during the eighth century, when the caliph al-Manṣūr (A.D. 754-775) obtained a copy as the result of a mission to the Byzantine court. A second Greek copy reached the Islamic world in the time of the caliph al-Ma'mūn (A.D. 813-833), again as the result of a diplomatic mission to the Byzantines [Heath 1905, 1, 75]. Thus this seminal work was known in the Islamic world from the time when the Islamic intellectual tradition was just beginning to take shape.

The first translations of the *Elements* were made by al-Ḥajjāj ibn Yūsuf ibn Maṭar, as we are told by Ibn al-Nadīm in his *Fihrist*. The first, made under the patronage of the caliph Hārūn al-Rashīd

(A.D. 786-809), was called the Ḥārūnī version. Al-Ḥajjāj translated the text again during the caliphate of al-Ma'mūn, and this was called the Ma'mūnī version [Heath 1905, 1, 75]. The first of these two translations appears to be completely lost. Parts of the second may exist in several other treatises. The most extensive is the commentary of al-Nayrīzī, which now exists only in an incomplete unique copy (Leiden 399, 1) containing books I-VI and a few lines of book VII. This commentary purports to be based directly on the second version of al-Ḥajjāj. A preface of unknown origin explains that this version is the result of a reediting or retranslating of the earlier version that al-Ḥajjāj had prepared at the behest of Yaḥyā ibn Khālid ibn Barmak, the vizier of Ḥārūn al-Rashīd. This revision represents his attempt, we are told, to leave out the superfluities, fill up the gaps, correct and remove errors, and generally to reduce the text to smaller dimensions without altering its substance [Heath 1905, 1, 75].

Engroff has recently discovered extracts of a Ḥajjāj version within Escurial ms. arabe 907. These extracts prove to be written in a style very different from that of the presumed Ḥajjāj extracts contained in the commentary of al-Nayrīzī. The most striking feature of these new extracts is that they lack nearly all the "helping phrases" which are found in the Nayrīzī text and which refer to earlier theorems or postulates. The style of the Nayrīzī text is unlike that of the Greek versions, the Arabic translation of Isḥāq ibn Ḥunayn, or any of the other Arabic commentaries on the *Elements*. Therefore, Engroff has argued that the commentary of al-Nayrīzī does not contain a Ḥajjāj translation in a pristine form but only a version heavily edited by al-Nayrīzī [Engroff 1980, 13-19].

A third Arabic translation, according to the *Fihrist*, was made by Isḥāq ibn Ḥunayn (d. A.D. 910), the son of the famous translator, Ḥunayn ibn Isḥāq [Heath 1905, 1, 75]. This translation of Isḥāq was revised by the mathematician, Thābit ibn Qurrah, who clearly used additional Greek manuscripts in his revision. This is indicated by the comment between propositions 30 and 31 of book IX [Escurial ms. arabe 907, f. 93a; Uppsala University, O. Vet. 20, f. 100b], where Thābit reports that he did not find these two propositions in the Greek manuscripts he had consulted. (Engroff's analysis of all the comments attributed to Thābit within the Arabic text of the *Elements* shows--surprisingly--that they are editorial, rather than mathematical, in content [Engroff 1980, 37-39].)

The outline by Heath summarized the state of our knowledge of the Arabic tradition of the *Elements* as recently as the middle of this century. The recent studies on the text of the Arabic versions of the *Elements*, while not yet dispelling the uncertainties about the tradition, have raised several questions about this received history: the most important concerns which branches of this complicated transmission may be represented in the extant Arabic manuscripts of the *Elements*. Of the twelve manuscripts

that are readily accessible, eleven either are explicitly attributed to the translation of Isḥāq (as revised by Thābit) or are so similar to such explicitly Isḥāq-Thābit texts that they are undoubtedly part of that tradition. The twelfth, Leningrad, Akademia Nauk, ms. C 2145, does not give the name of the translator, and displays some major variations from the Isḥāq-Thābit manuscripts. (This manuscript will be analyzed further at the conclusion of this article.)

Since the Isḥāq-Thābit version supposedly constitutes a single translation tradition, it is surprising to find two distinct textual families represented in these eleven manuscripts. These two families of texts may be distinguished by differences in vocabulary and style, as well as by variations in the order of the definitions and propositions between the two. Even more striking is the interpolation of two definitions into one of these textual families at the beginning of book VII. Of the ten manuscripts used in these studies, six belong to what may be termed Group A and four belong to Group B. Each of these families may, in turn, be further divided into two subfamilies. Within Group A, the mss. Copenhagen LXXXI and Istanbul, Fātiḥ 3439/1 compose one subfamily, and the mss. Tehran, Majlis Shūra 200; Rampur, ᶜArshī 200; and Chester Beatty 3035 form a second subfamily. Escurial ms. arabe 907 is a third element within Group A, and is especially important for the extracts of a Ḥajjāj translation that it contains. Group B contains two subfamilies as well. The first is made up of Oxford, Bod. Lib., Thurston 11 and Uppsala University, O. Vet. 20. The second is composed of Oxford, Bod. Lib., Huntington 435 and Cambridge University, add. 1035. The designation of the two families of manuscripts as Group A and Group B is completely arbitrary and does not indicate a judgement about the authenticity or accuracy of the different manuscript traditions. It only describes the observed variations among the manuscripts.

The recent study of books VII-IX [De Young 1981] clearly illustrates these textual differences. The differences in vocabulary between these two textual traditions are, of course, evident on first reading. Group A uses the term *al-akthar* (sometimes *al-akbar*) to mean "the larger," while Group B uses *al-aᶜẓam*. Similarly, Group A uses *al-aqall* to mean "the smaller," while Group B uses *al-aṣghar*. Of much greater interest, however, is the terminology introduced in the interpolated definitions 15 and 16 of book VII in the Group A manuscripts. These two definitions substitute the terms *mutabayyin* (mutually incommensurable) and *mushtarik* (commensurable) for the phrases *awwal ᶜinda al-ākhar* (prime to each other) and *murakkab ᶜinda al-ākhar* (composite with respect to each other), which are found in definitions 12 and 14 of the Greek text and in definitions 13 and 14 of both Group A and Group B.

It is not known why this new terminology was introduced. Was it, perhaps, to bring the terminology of books VII-IX into greater

conformity with that of book X (where Euclid discusses incommensurability at great length)? Given the great amount of attention focused on book X within the Arabic world, this is an appealing hypothesis. This hypothesis is made more appealing by the fact that each of the six recensions of the *Elements* examined in the study of these arithmetical books relies exclusively on the terminology of these interpolated definitions throughout their treatment of books VII-IX, regardless of the manuscript tradition to which they may seem, on other internal evidence, to be related.

It is also striking that in two of the manuscripts of Group A the definitions 13 and 14, which are the equivalent of the Greek definitions 12 and 14, are preceded by the phrase *min iṣlāḥ Thābit* (in the revision of Thābit), indicating, presumably, that they had been added by Thābit and were not present in the original Isḥāq translation [Istanbul, Fātiḥ 3439/1, f. 11b; Copenhagen LXXXI, f. 36a]. They are completely omitted from two other manuscripts [Escurial ms. arabe 907, f. 68a; Leningrad, Akademia Nauk ms. C 2145, f. 217a]. These definitions, although present in the rest of Group A and in Group B, are not there introduced by this qualifying phrase. In light of the connection Engroff has shown to exist between Escurial ms. arabe 907 and the Ḥajjāj tradition, we may speculate that perhaps these two definitions were omitted from the Ḥajjāj translations. We may also recall Klamroth's theory that Isḥaq merely adopted the definitions from the Ḥajjāj translation without retranslating them [Klamroth 1881, 310-311]. Although Engroff has shown that Klamroth's original thesis was based on insufficient and misinterpreted evidence, there may be some merit in reconsidering it in the light of more recent findings [Engroff 1980, 8-9].

If these definitions were missing from the work of al-Ḥajjāj, was this because they were missing from the Greek manuscripts on which he relied? None of the extant Greek manuscripts used by Heiberg contains such an omission, so this hypothesis appears unlikely. A more likely hypothesis is that these two definitions were omitted in favor of the interpolated definitions 15 and 16 of Group A in order to bring this arithmetical portion of the *Elements* into closer agreement with the terminology used in book X to discuss incommensurable magnitudes. On the other hand, we must note that only in the Leningrad manuscript does the terminology in book X match the terminology of these interpolated definitions in book VII. (All other manuscripts use *mushtarik* and *ghayr mushtarik*.) But since all other manuscripts seem to follow Group B formulation after proposition VIII-21 (discussed in detail below), and since Group B did not include the interpolated definitions, this strengthens, rather than weakens, the hypothesis. Given the association between the Escurial and Leningrad manuscripts and the Ḥajjāj tradition (discussed below), this seems to imply that this departure from the Greek version stems from the work of al-Ḥajjāj, who, we have been told, did not hesitate to "rewrite" Euclid--especially in his second translation of the *Elements*.

Although each of the six recensions of the *Elements* examined in the study of books VII-IX relies entirely on the terminology of these interpolated definitions, they do not at the same time omit definitions 13 and 14. Ibn Sīnā, in his *Shifā'*, includes definition 13, but not definition 14. The *Taḥrīr* of Muḥyi al-Dīn al-Maghribī, which is similar in style to the *Shifā'*, also omits definition 14. The *Taḥrīr* of Athīr al-Dīn al-Mufaḍḍal ibn ᶜUmar al-Abharī, on the other hand, omits definition 13 but includes definition 14. Naṣīr al-Dīn al-Ṭūsī, in his *Taḥrīr*, includes both definitions and ascribes them to Thābit. The Pseudo-Ṭūsī *Taḥrīr* [Sabra 1969, 18], printed in Rome in 1594, also contains both definitions. Each of these recensions also includes the interpolated definitions 15 and 16, except for those of al-Maghribī and Pseudo-Ṭūsī, which include only definition 15. These recensions were arbitrarily chosen so as to provide a random sample of the commentarial literature. The fact that each uses the terminology of Group A, definitions 15 and 16, although some have no other obvious resemblance to Group A, adds credence to the hypothesis that this new terminology may have been introduced to make books VII-IX more compatible with the discussion of incommensurability in book X. This appears likely, too, when we find that only the version of Pseudo-Ṭūsī retains the terminology of Groups A and B in book X. The remainder use the terminology of the Leningrad manuscript.

The variations in ordering of definitions and propositions in the various textual families when compared with the Greek text established by Heiberg are indicated in the tables of correspondence in the Appendix. This is not the place to point out every possible ramification of these variations--that should await further studies on the Euclidean tradition in the Arabic world. There are, however, two important features of the textual tradition that are not apparent from these tables. The first is that the membership of Group A and Group B is not constant. At the beginning of book VII there are six members in Group A and four members in Group B, as outlined above. Book VIII, however, opens with only the manuscripts Copenhagen LXXXI and Fātiḥ 3439/1, which make up one of the subfamilies of Group A, still present in Group A. The other manuscripts that had been in Group A join the Group B manuscript tradition until, with proposition 18 of book VIII, all merge to form a single textual tradition for the remainder of the arithmetical books. The simplest explanation for these changing alignments is that, at an early date, the prototype for the second subfamily of manuscripts in Group A was defective and the remainder of the text was supplied from a prototype for Group B. The prototype for the first Group A subfamily was more complete, and so had to be completed from the Group B tradition only after book VIII, proposition 18. This explanation, however, raises the question of what these prototypes were--the work of al-Ḥajjāj, Isḥāq, or an as-yet undiscovered editor? When the study of the entire text of the *Elements* is completed, we may be able to make some conjectures.

A second significant fact that is not revealed in these tables is that the order of propositions in the Leningrad, Akademia Nauk ms. C 2145 in many cases corresponds to the order of propositions found in the translation of al-Ḥajjāj (which Ḥajjāj version is not yet clear). This correspondence is inferred from comments by Naṣīr al-Dīn al-Ṭūsī in his *Taḥrīr* of the *Elements*. This valuable resource, which provides much indirect evidence about the translations of al-Ḥajjāj, includes the following comment after proposition 15 of book VIII:

> *And concerning the order of some of these propositions, it is different from what was presented to us according to the ordering of Thābit. As for al-Ḥajjāj, he presented to us what was given in propositions 11 and 12 [by Thābit] in proposition 11 alone; and he presented to us as proposition 13 what was presented as proposition 2 [by Thābit]; and there are presented in [al-Ḥajjāj] as propositions 13 and 14 [an error in the manuscript for propositions 12 and 14] the proofs presented [by Thābit] in propositions 14 and 15; and proposition 15 [of al-Ḥajjāj] is lacking [from Thābit's version]. After that, the two of them are in agreement.*

These statements are corroborated in the margin of one of the manuscripts which Sabra used to edit the summary of the *Elements* included in Ibn Sīnā's *Shifā'*:

> *What the shaykh discussed in the case of proposition 11 is, in the text of the* Elements *by Thābit, discussed in propositions 11 and 12; and what he [Thābit] discussed in proposition 2 is discussed in proposition 13. [The comment adds 14, but this appears to be an error--perhaps the scribe left out a line when he copied this note? Cf. Ṭūsī's comment.] What he [Thābit] discussed in the case of propositions 17 and 18 is discussed in the reverse order [by the shaykh]; and he [Thābit] introduced the converse of propositions 24 and 25 in two propositions like them. [Ṭūsī also notes this interpolation.] Thus the [number of the] propositions became 27. As for what the shaykh discussed, it is consistent with the text of al-Ḥajjāj.* [Ibn Sīnā 1976, 259]

Again, in book IX, Ṭūsī notes that the Ḥajjāj text interchanges propositions 11 and 12 (in relation to the Isḥāq-Thābit tradition) and moves proposition 14 to number 20. Each of these changes in the order corresponds to the order of propositions belonging to the Leningrad version, as indicated in the tables of correspondence in the Appendix.

Not only does the order of propositions in the Leningrad, Akademia Nauk ms. C 2145 usually correspond to these variations reported as the order of propositions in the Ḥajjāj versions, but we also find that propositions 20 and 21 of book VIII in the Leningrad manuscript correspond, with only minor variations, to two alternative proofs which follow VIII-21 in Escurial ms. arabe 907. There, these alternative proofs are explicitly attributed to al-Ḥajjāj. In the Leningrad manuscript we find only these Ḥajjāj proofs, but without an attribution to al-Ḥajjāj.

Propositions 20 and 21 of the Arabic versions correspond to propositions 22 and 23 of the Heiberg Greek edition of the *Elements*. Proposition 20 says "if three numbers be in continued proportion, and the first be square, the third will also be square." Proposition 21 says "if four numbers be in continued proportion, and the first be cube, the fourth will also be cube" [Heath 1926, 2,379]. The proofs of these two propositions in the Isḥāq-Thābit manuscript differ significantly from the proofs found in Heiberg's Greek edition. In order to illustrate these relationships, consider the translations of the Isḥāq-Thābit, Ḥajjāj, and Greek versions of these proofs.

In the Isḥāq-Thābit tradition we find the following proof for proposition 20:

> *The proof is that we take the smallest numbers [which are] according to the ratio of A, B, G, [and] whose number is as their number, namely D, E, Z. Thus the extremes, namely D, Z, are squares.*
>
> *Let the side of square A be number H, and the side of square D be number T, and the side of square Z be number K.*
>
> *Now, because the ratio of A, B, G is as the ratio of D, E, Z, and their number is as their number, they are in a ratio of equality, the ratio of A to G is as the ratio of D to Z.*
>
> *But each one of the numbers D, Z is prime to the other.*
>
> *Now numbers, some of which are prime to the others, are the smallest numbers [which are] according to their ratio. The smallest numbers [which are] according to their ratio measure numbers which are according to their ratio an equal number of times, the larger to the larger and the smaller to the smaller. Thus D measures A [by an amount] equal to that by which Z measures G.*
>
> *But if a square measures a square, its side measures its side. Thus T measures H.*
>
> *Let K measure L according to the amount by which T measures H. Thus the ratio of T to H is as the ratio of K to L.*

But the ratio of the square constructed from T to the square constructed from H is as the ratio of the square constructed from K to the square constructed from L.

Now, the square constructed from T is D, and the square constructed from H is A, and the square constructed from K is Z. Thus the ratio of D to A is as the ratio of Z to the square constructed from L.

But the ratio of D to A is as the ratio of Z to G. Thus the ratio of Z to G is as the ratio of Z to the square constructed from L.

Therefore, G is equal to the square constructed from L. Therefore, G is a square.

That is what we wanted to show.

This proof is completely different from that found in the Greek text, but the version ascribed to al-Ḥajjāj is much closer to the content and style of the Greek. It reads as follows:

The proof is that the ratio of A to B is as the ratio of B to G. Thus there falls between A and G a number, namely B, which is proportional to the two of them and they are all [continuously] proportional. Thus A and G are two similar plane numbers.

But A is a square.

Therefore, G is a square.

The proof in the Greek edition of Heiberg is even more condensed, although clearly formulated along the same lines:

Since there falls between A, G a mean proportional, [namely] B, then A, G are similar plane numbers.

But A is a square.

Therefore, G is a square. [cf. Heath 1905, 2, 379]

The proofs for proposition 21 (Greek proposition 23) follow the same pattern of argument, now using cubic numbers rather than squares.

Such major divergence from the Greek version of Heiberg is surprising. There are many minor variations between the Arabic versions and the Greek, but such extreme variations are very unusual. The origins of this "revised" version are unknown. Perhaps they reflect some corruption in the Greek texts used by the translators (although they do not correspond to variants given by Heiberg in his edition of the Greek). Perhaps they reflect the emendations of late Greek commentators, such as Hero. It seems clear that Thābit, at least, was cognizant of Hero's work, since Thābit is credited with introducing propositions VIII-23,24 which are identified by al-Nayrīzī as the work of Hero [Heath 1926, 2,

38; Ibn Sīnā 1976, 259]. Further studies of Arabic commentators and Arabic translations of Greek commentators are urgently needed for unraveling the process by which these variations entered the Arabic tradition.

The similarities between the proofs of propositions VIII-20, 21 in the Akademia Nauk ms. C 2145 and the alternative proofs from al-Ḥajjāj found in Escurial ms. arabe 907 certainly seem to imply that the former has some connection with the work of al-Ḥajjāj. It is equally clear, however, that this is not a pure Ḥajjāj version. Many of the comments of Thābit, which were analyzed by Engroff, are also present in the Leningrad manuscript, although not always with the attribution to Thābit. Moreover, book X, which is given its own title page in the Leningrad manuscript (unlike the other books in the same manuscript), is there specifically stated to be in the recension of Thābit ibn Qurrah, and at the end of book X there are two passages attributed to al-Ḥajjāj (implying that the remainder of the book is not his work). Therefore, although this manuscript has clear ties to the work of al-Ḥajjāj and at this moment probably represents the closest approach available to the Ḥajjāj tradition, the manuscript is not itself a Ḥajjāj text.

We are, then, left with two major puzzles in our attempt to understand the development of the Euclidean textual tradition in the Arabic world: (1) how do we come to have two different textual traditions within the Isḥāq-Thābit translation, and (2) what is the relationship between the Akademia Nauk ms. C 2145 and the versions of al-Ḥajjāj?

The standard understanding of the Arabic Euclidean tradition as found in Heath's study rests ultimately on the work of Ḥajjī Khalīfah and Ibn al-Nadīm. This received view of the tradition is inadequate, because the situation has been found to be more complex than this tradition would lead us to expect. A statement in Plooij [1950, 5] that Thābit ibn Qurrah had made two revisions of the *Elements*, the second better than the first, may help to explicate the situation. The source of this statement seems to be Kapp's study of the Euclidean tradition, which was based on the work of Ibn al-Qifṭī [Kapp 1935, 65]. Such a twofold revision by Thābit could go far toward explaining the difficulties inherent in the bifurcation within the Isḥāq-Thābit textual tradition, although it still does not reveal which of the extant manuscript traditions might represent the "better" version of Thābit's work. Perhaps Heath's sources have misled him on this matter.

With regard to the second outstanding question, consider the following remark by Ibn Abī Uṣaybiᶜah concerning al-Ḥajjāj: "He translated for al-Ma'mūn, and among his translations is the book of Euclid. Then Thābit ibn Qurrah al-Ḥarrānī revised his translation" [Ibn Abī Uṣaybiᶜah 1884, 1, 204]. Could it be that the Akademia Nauk ms. C 2145 represents part of a Ḥajjāj-Thābit tradition? Or is Ibn Abī Uṣaybiᶜah's statement incorrect--a garbled version of history?

Obviously, we are just beginning to untangle this complicated intellectual tradition, and can only offer conjectures about the interrelations of these various components until the Arabic versions of the *Elements* are available in a critical edition and further studies of commentaries within the Arabic tradition have been completed.

APPENDIX: TABLES OF CORRESPONDENCE

These tables of correspondence show how the order of definitions and propositions of each of our manuscript traditions corresponds to the order of definitions and propositions in the Greek text of Heiberg. In listing propositions, only places where orders differ are indicated. Note that numbers in square brackets refer to notes and not alternate numbering schemes.

Book VII Definitions

Heiberg	Group A	Group B	Leningrad
1	1	1	1
2	2	2	2
3	3	3	-
4	4	4	-
5	5	5	-
6	6	6	3
7	7	7	4
8	8	8	5
9	9	9	6
-	-	-	7
(9a)	-	-	-
10	10	10	8
11	11	11	9
12	13	13	12
13	12	12	-
14	14	14[1]	13
-	15	-	10
-	16	-	11
15	17	15	16
16	20	18	-
17	21	19	14
18	18	16	15
19	19	17	-
20	22	22	18
21	23[2]	20, 21	19
22	24	23	17

Book VII Propositions

Heiberg	Group A	Group B	Leningrad
7	7	10	7
8	8	11	8
9	9	7	9
10	10	8	10
11	11	12	11
12	12	13	12
13	13	9	13

Group A + Group B

21	22	21
22	21	22
29	31	31
30	32	32
31	29	29
32	30	30

Book VIII Propositions

Heiberg	Group A + Group B	Leningrad
1	1	1
2	2	13
Porism	Porism[3]	Porism
3	3	2
4	4	3
5	5	4
6	6	5
7	7	6
8	8	7
9	9	8
10	10	9
11	11	10
12	12	-
13	13	11
14	14	12
-	Porism[4]	-
15	15	14
-	Porism	-
16	-	15
17	-	(15a)[5]
18	16	16
19	17	17
20	18	17
21	19	19
22	20	20
23	21	21
24	22	22
25	23	23
-	24	-
-	25	-
26	26	24
27	27	25

Book IX Propositions

Heiberg	Group A + Group B	Leningrad
11	11	12
12	12	11
13	13	13
14	15	14
15	16	15
16	17	16
17	18	17
18	19	18
19	20	19
20	14	20
25	25	26
26	27	(24a)[6]
27	26	25
28	28	27
29	29	28
-	30	29
-	31	30
30	32	31
31	33	32
32	34	33
33	35	34
34	36	35
35	37	36
36	38	37

NOTES

1. Huntington 435 inserts the equivalent of the added definitions of Group A here.

2. Fātiḥ 3439/1 makes this two definitions, as do the Group B manuscripts.

3. This porism is missing from Escurial 907; Teheran, Majlis Shūra 200; Rampur, ᶜArshī 200; and Chester Beatty 3035.

4. This porism is missing from Teheran, Majlis Shūra 200; Rampur, ᶜArshī 200; and Chester Beatty 3035.

5. This is not treated as a separate proposition, but is incorporated into proposition 15.

6. This is not treated as a separate proposition, but is only a statement added in the margin of the manuscript.

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